ПРОЦЕССЫ УПРАВЛЕНИЯ

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Triaxial electrodynamic stabilization of a satellite via PID controller*

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A satellite moving on a circular equatorial orbit is considered. The satellite is equipped with a controlled magnetic moment and a controlled electrostatic charge. The problem of triaxial stabilization in the orbital frame is studied. The electrodynamic control system is proposed. In addition, to improve characteristics of transient processes, PID controller of a special form is used. With the aid of the Lyapunov direct method, asymptotic stability conditions of the program mode are obtained. The results of computer simulation are provided demonstrating the efficiency of the developed approach.

Keywords: satellite, electrodynamic control, triaxial stabilization, Lyapunov — Krasovskii functional, PID controller.

1. Introduction. The development of control systems for the attitude stabilization of satellites with respect to their centers of masses is one of the most actual and challenging problems of space dynamics [1–3]. There are various approaches to solving this problem, e.g., application of flywheels, power gyroscopes, reactive control systems, solar radiation pressure, etc. In last decades, magnetic control systems based on the interaction of their executive devices with the Earth magnetic field are widely used [4–9]. Such systems are especially effective for small satellites. Existing magnetic attitude control systems, their specific features, advantages and disadvantages are described, for instance, in [5, 10]. Moreover, it is worth noting that, to increase the efficiency of this approach, in [11], the electrodynamic method for the stabilization of satellites was proposed. This method is based on the simultaneous using the magnetic moment and the moment of Lorentz forces. In [12], the electrodynamic method was applied for the triaxial stabilization in the orbital

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frame for a satellite moving on a circular equatorial orbit. The problem of the satellite stabilization in the direct equilibrium position was solved.

However, it should be mentioned that, in many problems of satellite attitude control, it is necessary to ensure not only stabilization of program modes, but also sufficient smoothness of transient processes [13, 14]. An effective tool for damping undesirable oscillations of mechanical systems is the use of PID controllers and their modifications [15–19]. This method is widely applied in space dynamics (see, e.g., [20–22] and the bibliography cited therein). At the same time, in [17], it was noted that a general satisfactory theory and a general approach that can provide explicit design formulae for the PID parameters are still lacking.

The results of [12] were further extended in [23]. Firstly, instead of the direct equilibrium position, an arbitrary equilibrium position in the orbital frame was stabilized. Secondly, a development of the concept of the electrodynamic control was proposed providing compensation of disturbing torques. Thirdly, to improve the characteristics of transient processes, a modified PID controller was designed. The previously constructed control torque was supplemented by a term with distributed delay. Applying the Lyapunov direct method, conditions for the asymptotic stability of the program mode were derived. The effectiveness of the proposed control was confirmed by numerical modeling. It was shown that such an approach permits us not only to significantly smooth out transient processes, but also to increase the convergence rate to the program mode.

In the present contribution, we will propose another construction of a PID controller. It is worth mentioning that the controller designed in [23] is a special case of the new one. Conditions on the control parameters will be found under which the stabilization can be guaranteed. With the aid of numerical simulations, we will demonstrate that, via appropriate choice of these parameters, it is possible to improve the characteristics of transient processes compared with those obtained using the controller from [23].

2. Problem formulation. Consider a satellite equipped with a controlled magnetic moment I and a controlled electrostatic charge distributed over certain volume. Assume that its center of mass C moves on a circular equatorial low-Earth orbit. We will study the rotational motion of the satellite with respect to its center of mass in the orbital frame. Let the axis $C\xi$ of this frame be directed tangent to the orbit in the direction of motion, the axis $C\eta$ be directed normal to the orbital plane, and the axis $C\zeta$ be directed along the radius vector of the satellite's center of mass relative to the center of the Earth. Denote by ξ_0, η_0, ζ_0 the unite vectors of this frame. The orbital coordinate system rotates with respect to the inertial one with an angular velocity ω_0.

The system $Cxyz$ of principal central axes of inertia is rigidly connected with the satellite. In this system, the vectors ξ_0, η_0, ζ_0 are denoted by s_1, s_2, s_3. Assume that a right triple of mutually orthogonal unite vectors r_1, r_2, r_3 is given in the system $Cxyz$. These vectors define the program orientation of the satellite.

Let the perturbed gravitational torque M_G act on the satellite. As the satellite moves relative to the geomagnetic field with magnetic induction B, the Lorentz moment M_Λ and magnetic moment M_M are excited, where $M_\Lambda = P \times T$, $M_M = I \times B$, $P = \rho_0 Q$, Q is the total charge, ρ_0 is the radius vector of the satellite's charge center with respect to its center of mass, $T = V_C \times B$, V_C is the speed of the point C relative to the geomagnetic field. Assume that the vector B coincides with its value at the center of mass. In this paper, as well as in [12, 23], we will use "direct magnetic dipole" as a model of the geomagnetic field. In this case, the vector B is constant in all points of the equatorial orbit.

The rotation motion of the satellite is modeled by the Euler dynamic equations

$$J\frac{d\omega}{dt} - \omega_0 J(\omega \times s_2) + (\omega + \omega_0 s_2) \times (J(\omega + \omega_0 s_2)) = M_\Lambda + M_M + M_G \tag{1}$$

and the Poisson kinematic equations

$$\frac{ds_j}{dt} = -\omega \times s_j, \quad j = 1, 2, 3, \tag{2}$$

where ω is the angular velocity of the satellite with respect to the orbital frame; $J = \mathrm{diag}\{A_1, A_2, A_3\}$ is the inertia tensor in the coordinate system $Cxyz$; $M_G = 3\omega_0^2 s_3 \times (Js_3)$.

Our objective is to design an electrodynamic control providing triaxial stabilization of the prescribed program orientation of the satellite. This means that we should choose laws of variation of vectors I and P under which the system (1), (2) admits asymptotically stable equilibrium position

$$\omega = 0, \quad s_j = r_j, \quad j = 1, 2, 3. \tag{3}$$

3. Design of control torques. In [12], a special approach to solving the problem of a satellite triaxial stabilization in the direct equilibrium position was developed. Each of the vectors I and P was chosen as a sum of restoring and damping components. In [23], this approach was extended to the case of arbitrary equilibrium position in the orbital frame. Vectors I and P were supplemented by compensating components. The need for these additional terms is due to the fact that, under the action of disturbing forces, the considered system may not have the required program mode. As a result, the Euler equations were represented as follows:

$$J\frac{d\omega}{dt} - \omega_0 J(\omega \times s_2) + (\omega + \omega_0 s_2) \times (J\omega) + \omega_0 \omega \times (Js_2) =$$

$$= k_\Lambda r_3 \times s_3 + k_M r_2 \times s_2 - h_\Lambda(\omega - (\omega^\top s_3)s_3) - h_M(\omega - (\omega^\top s_2)s_2), \tag{4}$$

here $k_\Lambda, k_M, h_\Lambda, h_M$ are constant positive coefficients.

Furthermore, to improve the characteristics of transient processes (in particular, damping undesirable satellite oscillations), a term with distributed delay of the form

$$M_{\text{dist}} = ak_\Lambda \int_{t-\tau}^{t} r_3 \times s_3(u)du + ak_M \int_{t-\tau}^{t} r_2 \times s_2(u)du \tag{5}$$

was added to the right-hand side of (4). Here a and τ are constants with $\tau > 0$. Thus, a modified PID controller (see [16]) was constructed.

With the aid of the Lyapunov direct method, it was proven that if

$$\tau|a| < 1, \tag{6}$$

then there exist values of parameters $k_\Lambda, k_M, h_\Lambda, h_M$ under which the proposed controller ensures the asymptotic stability of the program mode (3). Moreover, the results of numerical simulations demonstrated that its use not only significantly smooth out transient processes, but also increases the convergence rate of perturbed motions to the equilibrium position.

In this paper, another type of PID controller will be applied. Instead of the integral term (5), we will add to the right-hand side of (4) the torque

$$\hat{M} = bk_\Lambda \int\limits_0^t f(t-u) r_3 \times s_3(u) du + bk_M \int\limits_0^t f(t-u) r_2 \times s_2(u) du, \tag{7}$$

where b is a constant coefficient; $f(t)$ is a nonnegative and piecewise continuous for $t \geqslant 0$ function such that $\int_0^\infty f(u)du < \infty$. Then the equations (4) take the form

$$J\frac{d\omega}{dt} - \omega_0 J(\omega \times s_2) + (\omega + \omega_0 s_2) \times (J\omega) + \omega_0 \omega \times (Js_2) = k_\Lambda r_3 \times s_3 +$$

$$+ k_M r_2 \times s_2 + bk_\Lambda \int\limits_0^t f(t-u) r_3 \times s_3(u) du + bk_M \int\limits_0^t f(t-u) r_2 \times s_2(u) du -$$

$$- h_\Lambda(\omega - (\omega^\top s_3)s_3) - h_M(\omega - (\omega^\top s_2)s_2). \tag{8}$$

Remark 1. The torque (5) can be obtained as a special case of (7) if $f(t) = a/b$ for $t \in [0, \tau]$ and $f(t) = 0$ for $t > \tau$.

We will look for conditions under which new controller ensures the triaxial stabilization of the satellite.

4. Stability analysis. Let the following additional constraints be imposed on the function $f(t)$.

Assumption 1. There exists a number $\lambda > 0$ such that $\int_t^\infty f(u)du \leqslant \lambda f(t)$ for $t \in [0, \infty)$.

Assumption 2. The inequality

$$|b| \int\limits_0^\infty f(u)du < 1 \tag{9}$$

holds.

Remark 2. The condition (9) is a counterpart of (6).

Remark 3. For instance, Assumption 1 is satisfied for the function $f(t) = \exp(-\delta t)$, where $\delta > 0$. It is worth noticing that PID controllers with exponential kernels are widely used in the contemporary control theory [24].

Denoting $h_\Lambda = h\hat{h}_\Lambda$, $h_M = h\hat{h}_M$, where $\hat{h}_\Lambda, \hat{h}_M$ are fixed positive constants and h is a positive parameter, we obtain

$$h_\Lambda(\omega - (\omega^\top s_3)s_3) + h_M(\omega - (\omega^\top s_2)s_2) = h\Xi\omega + hq(\omega, s_2, s_3),$$

here

$$\Xi = R \begin{pmatrix} \hat{h}_\Lambda + \hat{h}_M & 0 & 0 \\ 0 & \hat{h}_\Lambda & 0 \\ 0 & 0 & \hat{h}_M \end{pmatrix} R^\top,$$

the columns of the matrix R are vectors r_1, r_2, r_3, the function $q(\omega, s_2, s_3)$ satisfies the estimate $\|q(\omega, s_2, s_3)\| \leqslant \hat{\beta}\|\omega\|(\|s_2 - r_2\| + \|s_3 - r_3\|)$, $\hat{\beta} = \text{const} > 0$, $\|\cdot\|$ is the Euclidean norm of a vector. Thus, the matrix Ξ is positive definite.

Theorem. *Let Assumptions 1 and 2 be fulfilled. Then there exists a number* $\hat{h} > 0$ *such that the equilibrium position* (3) *of the system* (2), (8) *is asymptotically stable for all* $h \geqslant \hat{h}$.

P r o o f. Our analysis is based on the Lyapunov direct method and approaches to constructing Lyapunov functions and Lyapunov—Krasovskii functionals for mechanical systems developed in [25–28]. First, choose a Lyapunov function candidate as follows:

$$V(\omega, s_2, s_3) = \frac{1}{2}\left(k_\Lambda \|r_3 - s_3\|^2 + k_M \|r_2 - s_2\|^2\right) + \frac{1}{2}\mu\omega^\top J\omega - \frac{1}{h}\Phi^\top \Xi^{-1} J\omega.$$

Here $\Phi = k_\Lambda r_3 \times s_3 + k_M r_2 \times s_2$ and μ is a positive parameter to be selected later. Then

$$\frac{1}{2}\left(k_\Lambda \|r_3 - s_3\|^2 + k_M \|r_2 - s_2\|^2\right) + \mu\beta_1\|\omega\|^2 - \frac{1}{h}\beta_2\|\omega\|(\|r_3 - s_3\| + \|r_2 - s_2\|) \leqslant$$

$$\leqslant V(\omega, s_2, s_3) \leqslant \frac{1}{2}\left(k_\Lambda \|r_3 - s_3\|^2 + k_M \|r_2 - s_2\|^2\right) +$$

$$+ \mu\beta_3\|\omega\|^2 + \frac{1}{h}\beta_2\|\omega\|(\|r_3 - s_3\| + \|r_2 - s_2\|),$$

where $\beta_1, \beta_2, \beta_3$ are positive coefficients.

Differentiating the function $V(\omega, s_2, s_3)$ with respect to the system (2), (8), we obtain

$$\frac{dV}{dt} = \mu\omega^\top(t)\left(\Phi(t) + b\int_0^t f(t-u)\Phi(u)du - h\Xi\omega(t) - hq(\omega(t), s_2(t), s_3(t))\right) +$$

$$+ \frac{1}{h}(k_\Lambda r_3 \times (\omega(t) \times s_3(t)) + k_M r_2 \times (\omega(t) \times s_2(t)))^\top \Xi^{-1} J\omega(t) -$$

$$- \frac{1}{h}\Phi^\top \Xi^{-1}\left(\omega_0 J(\omega(t) \times s_2(t)) - (\omega(t) + \omega_0 s_2(t)) \times (J\omega(t)) - \omega_0\omega(t) \times (Js_2(t)) +\right.$$

$$\left. + \Phi(t) + b\int_0^t f(t-u)\Phi(u)du - hq(\omega(t), s_2(t), s_3(t))\right) \leqslant$$

$$\leqslant \mu\beta_4\|\omega(t)\|(\|r_3 - s_3(t)\| + \|r_2 - s_2(t)\|) + \mu|b|\|\omega(t)\|\int_0^t f(t-u)\|\Phi(u)\|du -$$

$$- h\mu\beta_5\|\omega(t)\|^2 - \frac{1}{h}\Phi^\top \Xi^{-1}\left(\Phi(t) + b\int_0^t f(t-u)\Phi(u)du\right) +$$

$$+ h\mu\beta_6\|\omega(t)\|^2(\|r_3 - s_3(t)\| + \|r_2 - s_2(t)\|) +$$

$$+ \frac{1}{h}\beta_7\|\omega(t)\|^2 + \frac{1}{h}\beta_8\left(\|\omega(t)\| + \|\omega(t)\|^2\right)(\|r_3 - s_3(t)\| + \|r_2 - s_2(t)\|) +$$

$$+ \beta_9\|\omega(t)\|(\|r_3 - s_3(t)\| + \|r_2 - s_2(t)\|)^2,$$

here $\beta_i > 0$, $i = 4, \dots, 9$.

Next, construct a Lyapunov — Krasovskii functional candidate in the form

$$W(t,\omega(t),s_{2t},s_{3t}) = V(\omega(t),s_2(t),s_3(t)) + \frac{1}{h}\int\limits_0^t \int\limits_{t-u}^{\infty} f(\sigma)d\sigma\Phi^\top(u)\Gamma\Phi(u)du,$$

where Γ is a constant positive definite matrix, s_{2t}, s_{3t} denote the restrictions of the corresponding components of a solution, i.e., $s_{lt}:\ u\mapsto s_l(u)$ for $u\in[0,t]$, $l=2,3$. Then

$$\frac{1}{2}\left(k_\Lambda\|r_3-s_3(t)\|^2 + k_M\|r_2-s_2(t)\|^2\right) - \frac{1}{h}\beta_2\|\omega(t)\|(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) +$$

$$+\mu\beta_1\|\omega(t)\|^2 + \frac{1}{h}\beta_{10}\int\limits_0^t\int\limits_{t-u}^{\infty} f(\sigma)d\sigma\|\Phi(u)\|^2du \leqslant W(t,\omega(t),s_{2t},s_{3t}) \leqslant$$

$$\leqslant \frac{1}{2}\left(k_\Lambda\|r_3-s_3(t)\|^2 + k_M\|r_2-s_2(t)\|^2\right) + \frac{1}{h}\beta_2\|\omega(t)\|(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) +$$

$$+\mu\beta_3\|\omega(t)\|^2 + \frac{1}{h}\beta_{11}\int\limits_0^t\int\limits_{t-u}^{\infty} f(\sigma)d\sigma\|\Phi(u)\|^2du,$$

$$\frac{dW}{dt} \leqslant \mu\beta_4\|\omega(t)\|(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) + \mu|b|\|\omega(t)\|\int\limits_0^t f(t-u)\|\Phi(u)\|du -$$

$$-h\mu\beta_5\|\omega(t)\|^2 + \frac{1}{h}\beta_8\left(\|\omega(t)\|+\|\omega(t)\|^2\right)(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) +$$

$$+h\mu\beta_6\|\omega(t)\|^2(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) + \frac{1}{h}\beta_7\|\omega(t)\|^2 +$$

$$+\beta_9\|\omega(t)\|(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|)^2 - \frac{1}{h}\Omega(t),$$

where β_{10},β_{11} are positive constants and

$$\Omega(t) = \Phi^\top\Xi^{-1}\left(\Phi(t) + b\int\limits_0^t f(t-u)\Phi(u)du\right) - \int\limits_0^\infty f(u)du\Phi^\top(t)\Gamma\Phi(t) +$$

$$+\int\limits_0^t f(t-u)\Phi^\top(u)\Gamma\Phi(u)du.$$

Let $Z(t)=\Xi^{-1/2}\Phi(t)$, $\gamma=\int_0^\infty f(u)du$. Then

$$\Omega(t) = \|Z(t)\|^2 + bZ^\top(t)\int\limits_0^t f(t-u)Z(u)du - \gamma Z^\top(t)\Xi^{1/2}\Gamma\Xi^{1/2}Z(t) +$$

$$+\int\limits_0^t f(t-u)Z^\top(u)\Xi^{1/2}\Gamma\Xi^{1/2}Z(u)du.$$

Choosing the matrix Γ in the form $\Gamma = \varepsilon \Xi^{-1}$, $\varepsilon = \text{const} > 0$, we obtain

$$\Omega(t) = (1-\varepsilon\gamma)\|Z(t)\|^2 + bZ^\top(t)\int\limits_0^t f(t-u)Z(u)du + \varepsilon\int\limits_0^t f(t-u)\|Z(u)\|^2du \geqslant$$

$$\geqslant \left(1-\varepsilon\gamma-\gamma\frac{|b|}{2\upsilon}\right)\|Z(t)\|^2 + \left(\varepsilon - \frac{|b|\upsilon}{2}\right)\int\limits_0^t f(t-u)\|Z(u)\|^2du,$$

where υ is an arbitrary positive number.

Let

$$\varepsilon\gamma + \gamma\frac{|b|}{2\upsilon} < 1, \quad \varepsilon > \frac{|b|\upsilon}{2}. \tag{10}$$

For the existence of $\upsilon > 0$ for which the inequalities (10) hold, it is necessary and sufficient the fulfilment of the condition

$$\gamma b^2 < 4\varepsilon(1-\gamma\varepsilon). \tag{11}$$

It is worth noticing that (11) imposes least conservative constraint on the parameter b in the case, where $\varepsilon = 1/(2\gamma)$. As a result, we arrive at the inequality (9).

For such a choice of parameters, the estimate of the derivative of $W(t,\omega(t),s_{2t},s_{3t})$ takes the form

$$\frac{dW}{dt} \leqslant \mu\beta_4\|\omega(t)\|(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) + \mu|b|\|\omega(t)\|\int\limits_0^t f(t-u)\|\Phi(u)\|du -$$

$$- h\mu\beta_5\|\omega(t)\|^2 + \frac{1}{h}\beta_7\|\omega(t)\|^2 + \frac{1}{h}\beta_8\left(\|\omega(t)\|+\|\omega(t)\|^2\right)(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) +$$

$$+ h\mu\beta_6\|\omega(t)\|^2(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|) + \beta_9\|\omega(t)\|(\|r_3-s_3(t)\|+\|r_2-s_2(t)\|)^2 -$$

$$- \frac{1}{h}\beta_{12}\|\Phi(t)\|^2 - \frac{1}{h}\beta_{13}\int\limits_0^t f(t-u)\|\Phi(u)\|^2du,$$

where $\beta_{12} > 0$, $\beta_{13} > 0$.

In [29], it was proven the existence of a number $\Delta > 0$ such that

$$\|\Phi(t)\|^2 \geqslant \frac{1}{2}\left(k_M^2\|r_2-s_2(t)\|^2 + k_\Lambda^2\|r_3-s_3(t)\|^2\right) \tag{12}$$

for $\|r_3-s_3(t)\|+\|r_2-s_2(t)\| < \Delta$. With the aid of the estimate (12) and the standard technique based on the application of Young's and Holder's inequalities (see [26, 27]), it is easy to verify that if values of μ and Δ are sufficiently small, then one can choose $\hat{h} > 0$ such that

$$\frac{1}{4}\left(k_\Lambda\|r_3-s_3(t)\|^2 + k_M\|r_2-s_2(t)\|^2\right) + \frac{1}{2}\mu\beta_1\|\omega(t)\|^2 +$$

$$+ \frac{1}{h}\beta_{14}\int\limits_0^t\int\limits_{t-u}^\infty f(\sigma)d\sigma(\|r_3-s_3(u)\|^2+\|r_2-s_2(u)\|^2)du \leqslant W(t,\omega(t),s_{2t},s_{3t}) \leqslant$$

$$\leqslant k_\Lambda\|r_3-s_3(t)\|^2 + k_M\|r_2-s_2(t)\|^2 + 2\mu\beta_3\|\omega(t)\|^2 +$$

$$+\frac{1}{h}\beta_{15}\int_0^t\int_{t-u}^{\infty} f(\sigma)d\sigma(\|r_3-s_3(u)\|^2+\|r_2-s_2(u)\|^2)du,$$

$$\frac{dW}{dt}\leqslant -\frac{1}{2}h\mu\beta_5\|\omega(t)\| - \frac{1}{h}\beta_{16}(\|r_3-s_3(t)\|^2+\|r_2-s_2(t)\|^2) -$$

$$-\frac{1}{h}\beta_{17}\int_0^t f(t-u)(\|r_3-s_3(u)\|^2+\|r_2-s_2(u)\|^2)du$$

for $h \geqslant \hat{h}$, $\|r_3 - s_3(t)\| + \|r_2 - s_2(t)\| < \Delta$, $\|\omega(t)\| < \Delta$. Here β_{14}, β_{15}, β_{16}, β_{17} are positive constants. Hence, the differential inequality

$$\frac{dW}{dt}\leqslant -\bar{\beta}W(t,\omega(t),s_{2t},s_{3t}), \qquad \bar{\beta}=\text{const}>0,$$

holds for $\|r_3 - s_3(t)\| + \|r_2 - s_2(t)\| < \Delta$, $\|\omega(t)\| < \Delta$. This implies the asymptotic stability of the equilibrium position (3). The proof is completed. □

5. Results of the computer modeling. For simulation, we assume that $\omega_0 = 0.001$, $k_\Lambda = k_M = 1$, $h_\Lambda = h_M = 1$, $J = \text{diag}\{1000, 1300, 1700\}$ (all dimensional parameters are given in International System of Units). Consider stabilization of a satellite in the direct equilibrium position, i.e., $r_1 = (1, 0, 0)^\top$, $r_2 = (0, 1, 0)^\top$, $r_3 = (0, 0, 1)^\top$. In Figures 1 and 2 the dependence of the value $\psi = \|r_3 - s_3\| + \|r_2 - s_2\|$ on time is presented. The following initial conditions for solutions were chosen: $\omega(0) = 0$,

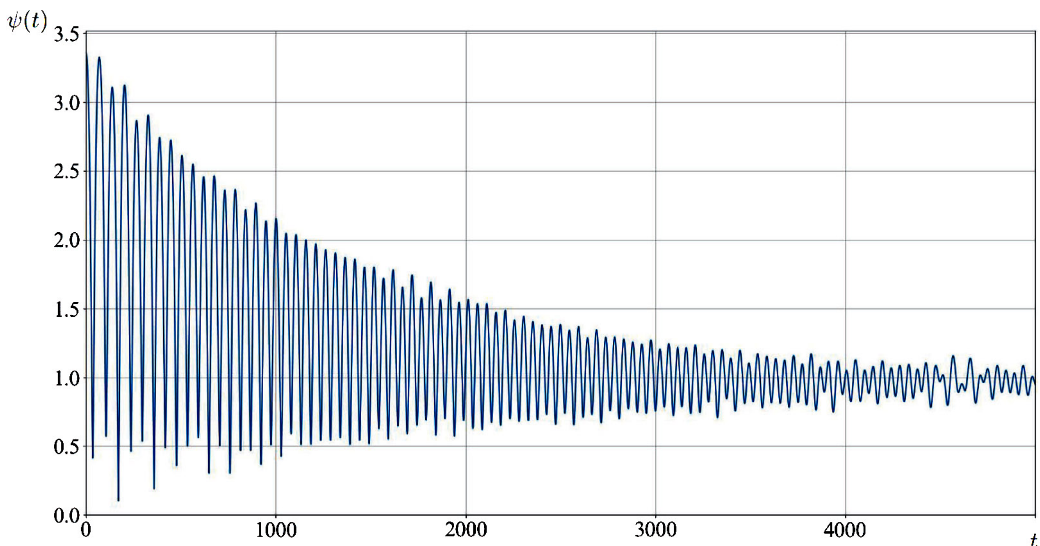

Figure 1. Stabilization process under control with distributed delay

$s_1(u) = (\cos\varphi, -\cos\theta\sin\varphi, \sin\theta\sin\varphi)^\top$, $s_2(u) = (\sin\varphi, \cos\theta\cos\varphi, -\sin\theta\cos\varphi)^\top$, $s_3(u) = (0, \sin\theta, \cos\theta)^\top$ for $u \leqslant 0$, where $\varphi = 0.3$, $\theta = -2$. Figure 1 corresponds to the application of the control with distributed delay. The term of the form (5) was used with the following values of parameters: $a = 1$, $\tau = 0.75$. In Figure 2, the result of application of PID controller is demonstrated. The integral term was defined by the formula (7) with $b = 3$, $f(t) = \exp(-4t)$. It is worth noticing that in this case $a\tau = b\int_0^\infty f(u)du = 0.75$.

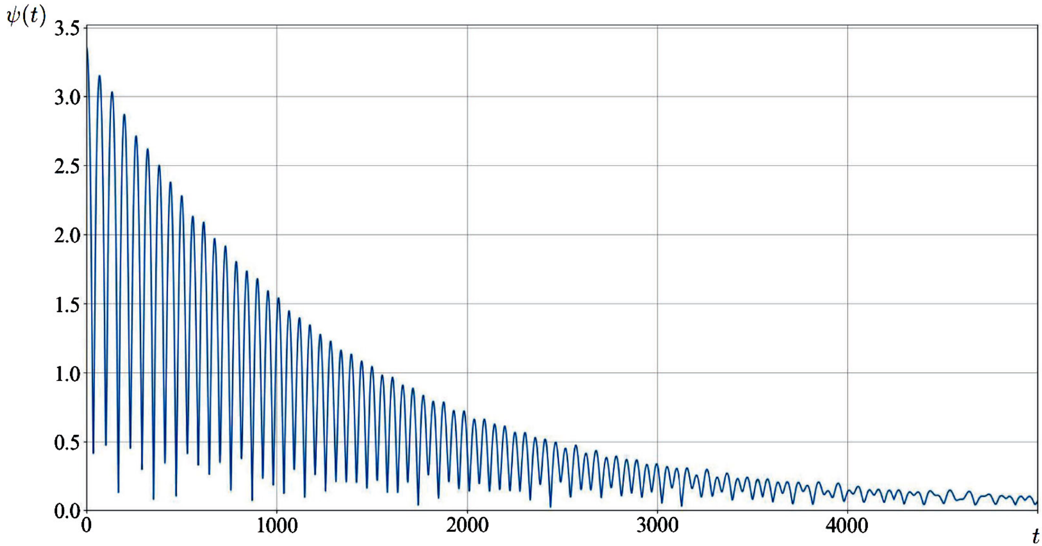

Figure 2. Stabilization process under PID controller

From the comparison of Figsures 1 and 2 it follows that, under the using new controller, transient processes become smoother and their time is significantly reduced.

6. Conclusion. In the present contribution, an electrodynamic control system ensuring triaxial stabilization of a satellite in the orbital frame is designed. The proposed controller includes an integral component that permits us to improve characteristics of transient processes. Using an original construction of Lyapunov — Krasovskii functional, conditions on control parameters are derived guaranteeing the asymptotic stability of the prescribed equilibrium position. It is worth noticing that these conditions contain some constraints on the differential and integral components of the controller, but do not impose any constraints on the proportional component. The effectiveness of the proposed approach is confirmed by numerical simulation. It is shown that, in comparison with the previously constructed control, the new one permits us not only to significantly smooth out transient processes, but also to increase the convergence rate to the program mode. An important direction for further research is an extension of the obtained results to the case of more complex model of the magnetic field.

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Трехосная электродинамическая стабилизация спутника с использованием ПИД-регулятора*

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Рассматривается спутник, движущийся по круговой экваториальной орбите. Спутник оснащен управляемым магнитным моментом и управляемым электростатическим зарядом. Исследована задача трехосной стабилизации спутника в орбитальной системе координат. Предложена электродинамическая система управления. Кроме того, для улучшения характеристик переходных процессов используется ПИД-регулятор специальной формы. С помощью прямого метода Ляпунова получены условия асимптотической устойчивости программного режима. Приведены результаты компьютерного моделирования, демонстрирующие эффективность разработанного подхода.

Ключевые слова: спутник, электродинамическое управление, трехосная стабилизация, функционал Ляпунова—Красовского, ПИД-регулятор.

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