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Determining the structure and parameters of the human model subjected to vibration

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This article presents results from an experimental study of the frequency properties of a sitting person. This analysis shows that in order to define the structure and parameters of a human body model, it is necessary to find the only solution to the problem of determining the model parameters. Otherwise, this model cannot be used to build vibration protection systems, which is its main purpose. This problem was solved for a two-mass human model. To do this, the total model mass and although the amplitude-frequency response and input frequency impedance for both solids in the model structure are needed to be known. In addition to this, the influence of multi-articular muscles on the frequency characteristics of a human body is found using mechanical models with an arbitrary number of degrees of freedom. In particular, the possibility of the existence additional antiresonance frequencies on the upper mass is shown.

Keywords: vibration, human body, mechanical model, frequency response, multiarticular muscle.

1. Introduction. A large number of papers have been devoted to the experimental determination of frequency properties of a human body, which can be represented by the amplitude-frequency response (AFR) or input frequency impedance (IMI). On their basis, mechanical models of the human body subject to vibration were built. The main purpose of these models is the construction of human protection systems against vibration.

The first researcher who measured the impedance of the human body was von Bekesi [1]. He used this frequency response to determine the limit of a person's sensitivity to vibration. The advent of more modern sensors for measuring force and acceleration has generated a stream of experimental studies of the human body frequency characteristics. Thus, in [2] the results of measuring the IMI of a sitting person in tense and relaxed positions with prolonged vibration in the frequency range from 0 to 20 Hz were presented. The graph clearly shows two resonant peaks in the region of 5–6 and 11–12 Hz for all postures. It was also noted that the behavior of the sitting human body impedance is similar to that of a mechanical system consisting of two rigid bodies connected in series by springs and dampers. Later papers repeatedly drew attention to the strong influence of the subject's posture on the transmission of vibration from the seat to the head [3, 4], as well as to a significant variability in the measurement results for the different volunteers [5, 6].

The first mechanical models of a human body exposed to vertical vibration appeared half a century ago [7]. These models were based on experimental studies for three different poses of subjects and had a structure consisting of sequentially arranged solid bodies connected by springs and dampers, but differed from each other in the number of degrees of freedom. The model parameters were selected so that the frequency response of the model was as close as possible to the experimental frequency response. In subsequent papers the structure of mechanical models became more complex. In [8] a lumped-parameter model of a human body in the sitting position is formulated. It includes the head, vertebral column, upper torso, abdomen-thorax viscera, pelvis and legs. The deformability of some of them is modeled by springs and dampers. The same authors proposed a mechanical model in which the number of degrees of freedom was reduced to three [9], and according to the authors, the reproducibility of experimental results deteriorated only slightly. A similar model with three degrees of freedom was presented in [10]. These examples show that the problem of determining the structure was not solved and the reasons for the resonance of the human body have not been fully studied [11].

The uncertainty of the structure of models is also a characteristic of recent works. Thus, in [12] a group of models with four degrees of freedom was presented, in which solid bodies were connected by springs and dampers in different combinations not only in series but also in parallel. While doing so, the model elements were not associated with specific parts of the human body. Therefore, the proposed models are not models of the human body, but only models of experimental AFR or IMI. This is also confirmed by models from [13–15]. In this regard, the number one problem in modeling the human body is to determine the structure of the mechanical model.

The second problem is the problem of determining a unique set of parameter values for a mechanical model of a human body. This is extremely important, because having two or more values for at least one parameter will make it impossible to be used for building a vibration protection system. Unfortunately, this problem has neither been resolved yet, nor has it been formulated.

Another problem is that the previously proposed mechanical models did not take into account and did not study the influence of multi-joint muscles on the frequency properties of a sitting human body. Although it is well known that they provide mobility in the cervical, thoracic and lumbar spine and maintain the equilibrium position.

These problems are the subject of the research in this article.

2. Principles of model construction and determination of its parameters. The problem of determining the model structure and determining the unique set of its parameters' values are interrelated. As stated in the introduction, in order to determine the structure of a model, all its elements modeled by solid bodies must correspond to certain parts of a human body. For all elements, the mass must be determined based on the average anatomical data. The sum of these masses must be equal to the mass of the human body per seat, which is determined by weighing. In addition, it is necessary to experimentally determine the transfer function from the seat to this simulated area of the human body for each solid body in the model. Such measurements are absent in the vast majority of previous works. A rare exception is the model presented in [16]. The disadvantage of this article is that vibration measurements were made not on the vertebrae themselves but on the skin area adjacent to the vertebra.

As for searching for a single set of model parameters' values, for clarity, we will consider a simple mechanical model of a chain structure with two degrees of freedom that is shown in Fig. 1. In it m_1 and m_2 are the upper and lower body mass, c_1 and c_2 are the spring stiffness, b_1 and b_2 are the damping coefficients and $y(t)$ is displacement of the model vibrating base.

The motion equations of the given model in the absolute coordinate system have the form

$$
\begin{aligned}
&m_1\ddot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + c_1(x_1 - x_2) = 0,\\
&m_2\ddot{x}_2 - b_1(\dot{x}_1 - \dot{x}_2) - c_1(x_1 - x_2) + b_2(\dot{x}_2 - \dot{y}) + c_2(x_2 - y) = 0.
\end{aligned}
\tag{1}
$$

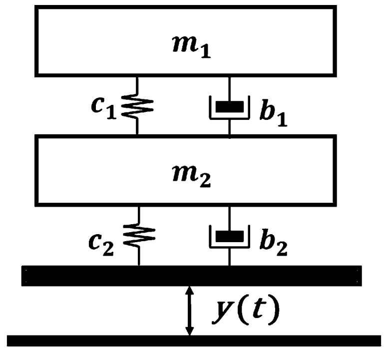

Fig. 1. Two-mass mechanical model of a human body subjected to vibration

Applying the Laplace transformation to the system of motion equations of (1) we can obtain the transfer functions for masses m_1 and m_2:

$$
H_1(p) = \frac{X_1(p)}{Y(p)} = \frac{\alpha_2 p^2 + \alpha_1 p + \alpha_0}{\delta_4 p^4 + \delta_3 p^3 + \delta_2 p^2 + \delta_1 p + \delta_0}\,, \tag{2}
$$

$$
H_2(p) = \frac{X_2(p)}{Y(p)} = \frac{\beta_3 p^3 + \beta_2 p^2 + \beta_1 p + \beta_0}{\delta_4 p^4 + \delta_3 p^3 + \delta_2 p^2 + \delta_1 p + \delta_0}\,. \tag{3}
$$

The coefficients in these transfer functions are related to the model parameters in the following way:

$$
\begin{gathered}
\delta_0 = \alpha_0 = \beta_0 = c_1 c_2\,, \quad \delta_1 = \alpha_1 = \beta_1 = c_1 b_2 + c_2 b_1\,,\\
\delta_2 = (m_1 + m_1)c_1 + m_1 c_2 + b_1 b_2\,, \quad \delta_3 = (m_1 + m_1)b_1 + m_1 b_2\,,\\
\delta_4 = m_1 m_2\,, \quad \alpha_2 = b_1 b_2\,, \quad \beta_2 = m_1 c_2 + b_1 b_2\,, \quad \beta_3 = m_1 b_2\,.
\end{gathered}
\tag{4}
$$

The AFR corresponding to the transfer functions (2) and (3) has the form

$$
|H_1(\omega)| = \sqrt{\frac{(\alpha_0 - \alpha_2\omega^2)^2 + (\alpha_1\omega)^2}{(\delta_4\omega^4 - \delta_2\omega^2 + \delta_0)^2 + (\delta_1\omega - \delta_3\omega^3)^2}}\,, \tag{5}
$$

$$
|H_2(\omega)| = \sqrt{\frac{(\beta_0 - \beta_2\omega^2)^2 + (\alpha_1\omega - \delta_3\omega^3)^2}{(\delta_4\omega^4 - \delta_2\omega^2 + \delta_0)^2 + (\beta_1\omega - \delta_3\omega^3)^2}}\,. \tag{6}
$$

The values of the coefficients δ_i, α_i and β_i are determined in a way to bring the theoretical frequency responses (5) and (6) as close as possible to the experimental ones. Having thus their values determined, in accordance with the relations (5) we will have eight equations, from which we can determine the values of six unknown parameters of the model m_1, m_2, c_1, c_2, b_1, b_2 provided that the total mass of the model is known $M = m_1 + m_2$:

$$
c_1 = \frac{\delta_2 - \beta_2}{M}\,, \quad b_1 = \frac{\delta_3 - \beta_3}{M}\,, \quad c_2 = \frac{M\delta_0}{\delta_2 - \beta_2} = \frac{M\alpha_0}{\delta_2 - \beta_2} = \frac{M\beta_0}{\delta_2 - \beta_2}\,,
$$

$$b_2 = \frac{M\alpha_2}{\delta_3 - \beta_3}\,, \quad m_1 = \frac{\beta_3(\delta_3 - \beta_3)}{M\alpha_2}\,, \quad m_2 = \frac{M\alpha_2\delta_4}{\beta_3\delta_3 - \beta_3}\,.$$

As you can see in this case, you do not need to pre-set the masses m_1 and m_2 based on the average anatomical data as it was required above to determine the structure of the model.

However, in most of the studies, only the experimental frequency response for mass m_1 is available to determine the values of the model parameters. In this case, only six equations out of of the eight (4) are remained. As a result, we will not be able to uniquely determine the values of all the model parameters, since for parameters m_1, c_1 and b_1 we will get square equations, namely

$$m_1{}^2 - Mm_1 + \delta_4 = 0, \quad b_2c_1{}^2 - \alpha_1c_1 + \alpha_0b_1 = 0, \quad Mb_1{}^2 - \delta_3b_1 + m_1\alpha_2 = 0.$$

As we can see, prior knowledge of the masses m_1 and m_2 does not save the situation.

For a mechanical model of a chain structure with three degrees of freedom similar results are obtained, but their proof is more cumbersome. An additional research is necessary for mechanical models of a chain structure with an arbitrary number of degrees of freedom.

In a number of works the IMI is used as an experimentally determined frequency characteristic of a human body.

The modulus of IMI for a mechanical model (Fig. 1) has the form

$$|Z(\omega)| = \sqrt{\frac{(f_3\omega^4 - f_1\omega^2)^2 + (f_0\omega - f_2\omega^3)^2}{(\delta_4\omega^4 - \delta_2\omega^2 + \delta_0)^2 + (\delta_1\omega - \delta_3\omega^3)^2}}\,,$$

where

$$f_0 = M\delta_0, \quad f_1 = M\delta_1, \quad f_2 = Mb_1b_2 + m_1m_2c_2, \quad f_3 = m_1m_2b_2,$$

δ_i are the same as in (4).

As for the values of f_i and δ_i, as indicated above, they are determined based on the condition of the maximum approximation of the theoretical module of the IMI to the experimental one. Having these values determined, we can proceed to defining the model parameters. In doing so, the unique set of their values we can get only if we know the value of one of the masses, for example m_1. Then we get:

$$c_1 = \frac{\delta_1m_1 - \delta_2b_1 + b_1{}^2b_2}{m_1b_2 + Mb_1}, \;\; c_2 = \frac{\delta_0}{c_2}, \;\; b_1 = \frac{\delta_3 - b_2m_1}{M}, \;\; b_2 = \frac{f_3}{\delta_4}, \;\; m_2 = M - m_1.$$

By the way, you can also determine the total mass of the model without weighing because $M = f_0/\delta_0$.

Thus, to uniquely determine the parameters of a two-mass model of a human body using IMI you need to know the value of one of the masses.

3. Features of the trunk muscles. The peculiarity of the trunk muscles is that most of them are multi-articular. It is known that these muscles largely determine the mechanical properties of a spine, and in their absence, the vertebral column is not able to maintain its configuration. However, the influence of multi-joint skeletal muscles on the mechanical characteristics of a human body under vibration has not been studied, including when modeling the human body exposed by vibration.

Since these muscles are in a constant tension under vibration conditions, they can be modeled by multi-link viscoelastic joints [17] as shown in Fig. 2 for the simplest human body model with two degrees of freedom.

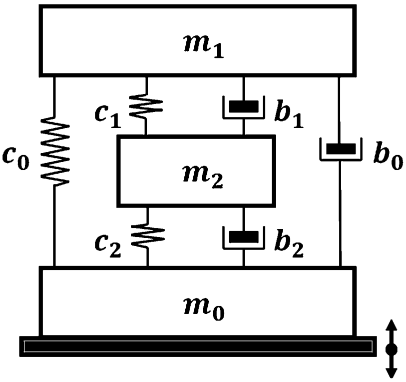

Fig. 2. Mechanical chain structure system with two degrees of freedom in the presence of two-link connections

The transfer functions for masses m_1 and m_2 are written:

$$H_1(p) = \frac{X_1(p)}{Y(p)} \frac{\alpha_3 p^3 + \alpha_2 p^2 + \alpha_1 p + \alpha_0}{\delta_4 p^4 + \delta_3 p^3 + \delta_2 p^2 + \delta_1 p + \delta_0},$$
$$H_2(p) = \frac{X_2(p)}{Y(p)} \frac{\beta_3 p^3 + \beta_2 p^2 + \beta_1 p + \beta_0}{\delta_4 p^4 + \delta_3 p^3 + \delta_2 p^2 + \delta_1 p + \delta_0},$$

where

$$\begin{aligned}
&\delta_0 = \alpha_0 = \beta_0 = c_1 c_2 + c_0(c_1 + c_2), \\
&\delta_1 = \alpha_1 = \beta_1 = c_1 b_2 + c_2 b_1 + b_0(c_1 + c_2) + c_0(b_1 + b_2), \\
&\delta_2 = c_1(m_1 + m_1) + c_2 m_1 + b_1 b_2 + b_0(b_1 + b_2) + c_0 m_2, \\
&\delta_3 = b_1(m_1 + m_1) + b_2 m_1 + b_0 m_2, \quad \delta_4 = m_1 m_2, \quad \alpha_2 = b_1 b_2 + c_0 m_2, \\
&\alpha_3 = b_0 m_2, \quad \beta_2 = m_1 c_2 + b_1 b_2 + b_0(b_1 + b_2), \quad \beta_3 = m_1 b_2.
\end{aligned}$$

The frequency responses for masses m_1 and m_2 are as follows:

$$|H_1(\omega)| = \sqrt{\frac{(\alpha_0 - \alpha_2\omega^2)^2 + (\alpha_1\omega)^2 - \alpha_3\omega)^3}{(\delta_4\omega^4 - \delta_2\omega^2 + \delta_0)^2 + (\delta_1\omega - \delta_3\omega^3)^2}},$$

$$|H_2(\omega)| = \sqrt{\frac{(\beta_0 - \beta_2\omega^2)^2 + (\alpha_1\omega - \delta_3\omega^3)^2}{(\delta_4\omega^4 - \delta_2\omega^2 + \delta_0)^2 + (\beta_1\omega - \delta_3\omega^3)^2}}.$$

It should be noted that the numerator of frequency response $|H_1(\omega)|$ for $b_j = 0$ is equal to $\alpha_0 - \alpha_2\omega^2$ and vanish, when

$$\omega = \sqrt{\frac{\alpha_0}{\alpha_2}} = \sqrt{\frac{c_1 c_2 + c_0(c_1 + c_2)}{c_0 m_2}}.$$

This means that when two-link connections are applied, an antiresonance frequency appears on the upper mass, but this is not possible in the absence of two-link connections. It should be noted that multi-link joints can bind different masses of the system, depending on which multi-joint muscles are included in the developed model. Consider, for example, the case of the existence of antiresonance frequencies on the upper mass of a model with three

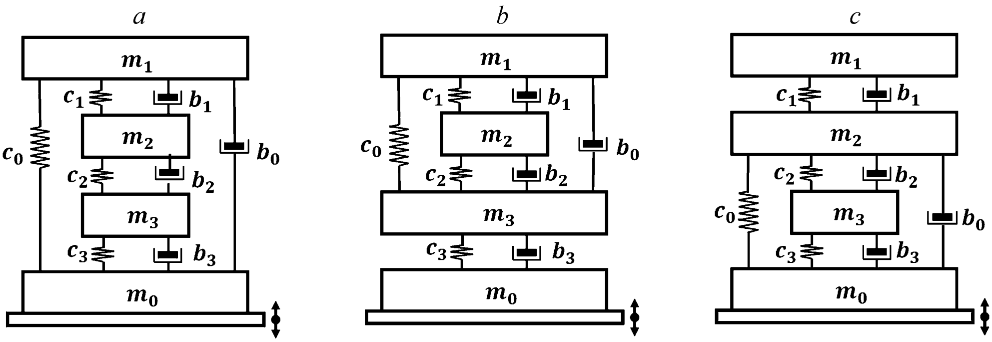

Fig. 3. Three variants (*a–c*) for applying multi-link connections to a model with three degrees of freedom

degrees of freedom, depending on how the additional multi-link connection is imposed. In this case, there are 3 variants for applying an additional connection (Fig. 3).

As an illustration, we have given numerical calculations of the AFR for the human body mechanical model with eight degrees of freedom (Fig. 4) in the presence (curve *2*) and absence (curve *1*) of multi-link connections. In this model m_1 is the mass of the head, m_0 is the mass of the trunk, and $m_2, ..., m_8$ are the masses of seven cervical vertebrae. The Fig. 4 shows that the presence of multi-link connections (curve *2*), which model multi-joint neck muscles, can lead to a decrease in the maximum values of the frequency response and the appearance of a frequency at which the AFR is close to zero.

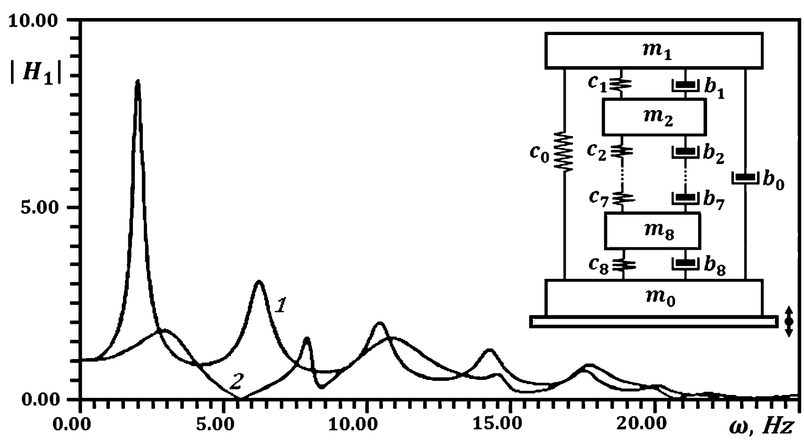

Fig. 4. AFR for the models with eight degrees of freedom

1 — the usual chain structure; *2* — the chain structure with multi-link connections

4. Conclusion. Thus, it is shown that in order for the mechanical model of the human body to be suitable for use in the construction of vibration protection systems, it is necessary to find a single set of numerical values of its parameters. Using the example of a two-mass mechanical model of the human body, it was shown that an unambiguous determination of its parameters is possible only if there are experimental frequency characteristics for both masses and the value of the total mass, or if there is an experimental IMI and the value of one of the masses.

In addition, the influence of multi-articular muscles on the frequency characteristics of the human body was found using mechanical models. In particular, the possibility of antiresonance frequencies, including at the upper mass, was shown.

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Определение структуры и параметров модели человека, подверженного вибрации

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В статье представлены результаты экспериментальных исследований частотных свойств сидящего человека. Их анализ показал, что для определения структуры и параметров модели человеческого тела необходимо найти единственное решение проблемы определения параметров модели. В противном случае эта модель не может быть использована для создания систем виброзащиты, что является ее основным назначением. Данная проблема была решена для двухмассовой модели человека. Для этого необходимо знать общую массу модели, а также амплитудно-частотную характеристику и входное частотное сопротивление для обоих твердых тел в структуре модели. Кроме того, было определено влияние многосуставных мышц на частотные характеристики человеческого тела с использованием механических моделей с произвольным числом степеней свободы. В частности, показана возможность существования дополнительных антирезонансных частот на верхней массе.

Ключевые слова: вибрация, тело человека, механическая модель, частотная характеристика, многосуставная мышца.

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