UDC 51-73, 537.2 Вестник СПбГУ. Прикладная математика. Информатика... 2023. Т. 19. Вып. 4
MSC 35J05

Field emission system with two emitters mathematical modeling*

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For citation: Egorov N. V., Vinogradova E. M., Kurbatova G. I., Klemeshev V. A. Field emission system with two emitters mathematical modeling. *Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes*, 2023, vol. 19, iss. 4, pp. 443–448.
https://doi.org/10.21638/11701/spbu10.2023.402

This paper presents a mathematical model of a two-dimensional emission diode system with two identical field emitters on a flat substrate; the anode is a plane parallel to the plane of the substrate. According to the proposed method, the real field cathode coincides with the virtual cathode, the shape of which is determined by the zero equipotential surface. The influence of each of the field emitters on the electrostatic potential distribution is replaced by the influence for a finite number of charged filaments. The solution of the boundary value problem for the Poisson equation is found in an analytical form. The potential distribution over the entire region of the emission system under study is presented in the form of expansions in terms of eigenfunctions. The coefficients of the series are calculated explicitly. The graphs presented demonstrate the dependence of the potential distribution on the distance between the emitters. All geometric dimensions of the system are the parameters of the problem.

Keywords: micro- and nanoelectronics, field emitter, mathematical modeling, electrostatic potential distribution, boundary-value problem.

1. Introduction. Field cathodes are widely used in a variety of modern vacuum electronic devices [1–3]. The field cathode is structurally a thin emitter with a radius of curvature at its top, on the order of several micro- or nanometers [4–6]. This shape of the emitter makes it possible to obtain intense field electron emission at relatively low voltages [7, 8]. However, one of the main disadvantages of field cathodes is that they give small values of the total emission current for single emitter — on the order of several microamperes. This work is devoted to the mathematical modeling of a two-dimensional emission diode system with two identical field emitters on a flat substrate. The anode is a plane parallel to the plane of the substrate. Two field emitters schematic representation in Cartesian coordinates (x, y) is shown in Figure 1. The influence of each field emitter on the potential distribution can be replaced by the influence of N charged filaments with charge densities τ_i located in the xOy plane with coordinates $(x = x_0,\ y = y_{0_i})$ and $(x = x_2 - x_0,\ y = y_{0_i})$, $i = \overline{1, N}$. The surface of the real cathode coincides with the zero equipotential [9–12].

2. Mathematical model. The problem parameters: L — the height of the emitters; $y = 0$, $0 \leqslant x \leqslant x_2$ — the surface of the emitter's substrate; $y = y_1$, $0 \leqslant x \leqslant x_2$ — the surface of the anode; $x = 0$, $x = x_2$, $0 \leqslant y \leqslant y_1$ — the boundaries of the domain along the x axis; $x = x_1 = x_2/2$, $0 \leqslant y \leqslant y_1$ — the plane of symmetry of the system.

The boundary conditions in an emission diode system with two field emitters: $U(x, 0) = 0$, $0 \leqslant x \leqslant x_2$, — on the substrate surface; $U(x, y_1) = U_0$, $0 \leqslant x \leqslant x_2$, —

* This work was funded by the Russian Science Foundation (project N 23-29-00937, https://rscf.ru/project/23-29-00937/).

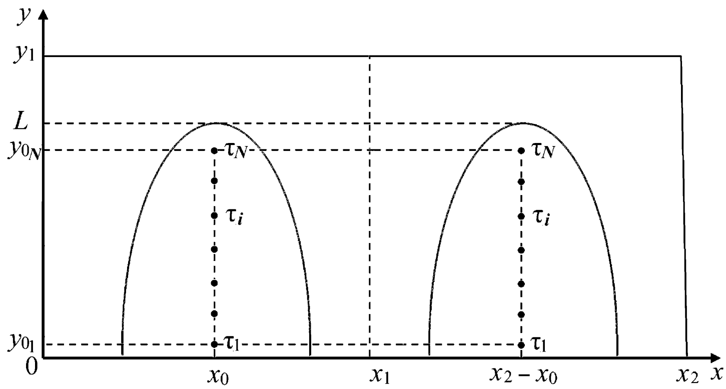

Figure 1. Schematic representation of the diode field emission system with two identical field emitters

on the anode surface; $U(0, y) = U(x_2, y) = U_0 \dfrac{y}{y_1}$, $0 \leqslant y \leqslant y_1$, — the boundaries of the domain in terms of the variable y.

The potential distribution function $U(x, y)$ for the entire region $0 \leqslant x \leqslant x_2$, $0 \leqslant y \leqslant y_1$ is symmetric with respect to plane $x = x_1$, $0 \leqslant y \leqslant y_1$, so the problem can be solved for $0 \leqslant x \leqslant x_1$, $0 \leqslant y \leqslant y_1$.

Let each charged filament with linear charge density τ_i create a uniformly distributed space charge ρ in a small volume $|x - x_0| < \varepsilon$, $|y - y_{0_i}| < \delta$. Thus the electrostatic potential distribution $U(x, y)$ is a solution of the Poisson's equation for the boundary-value problem:

$$\begin{cases} \dfrac{\partial^2 U(x,y)}{\partial x^2} + \dfrac{\partial^2 U(x,y)}{\partial y^2} = -\dfrac{1}{\varepsilon_0}\rho(x,y), \\ U(x,0) = 0, \quad 0 \leqslant x \leqslant x_1, \\ U(x,y_1) = U_0, \quad 0 \leqslant x \leqslant x_1, \\ U(0,y) = U_0 \dfrac{y}{y_1}, \quad 0 \leqslant y \leqslant y_1, \\ \left.\dfrac{\partial U(x,y)}{\partial x}\right|_{x=x_1} = 0, \quad 0 \leqslant y \leqslant y_1. \end{cases} \tag{1}$$

The function $\rho(x, y)$ on the right side of the Poisson equation for the boundary value problem (1) can be represented as

$$\rho(x,y) = \begin{cases} \rho_i, & |x - x_0| < \varepsilon \quad \text{and} \quad |y - y_{0_i}| < \delta, \\ 0, & |x - x_0| > \varepsilon \text{ or } |y - y_{0_i}| > \delta, \end{cases} \tag{2}$$

where the relationship between the space charge (2) and the linear charge density of the filament is given by the condition

$$\tau_i = \lim_{\varepsilon,\delta \to 0} 4\varepsilon\delta\,\rho_i, \quad i = \overline{1, N}. \tag{3}$$

The potential distribution $U(x, y)$ can be represented in the form

$$U(x,y) = U_0 \frac{y}{y_1} + U_1(x,y). \tag{4}$$

Then formulas (2)–(4) for the function $U(x,y)$ make it possible to satisfy all the boundary conditions of the problem (1) with respect to the variables x and y, if the function $U_1(x,y)$ is solution of the Poisson equation with homogeneous boundary conditions in both variables:

$$\begin{cases} \dfrac{\partial^2 U_1(x,y)}{\partial x^2} + \dfrac{\partial^2 U_1(x,y)}{\partial y^2} = -\dfrac{1}{\varepsilon_0}\rho(x,y), \\ U_1(x,0) = 0, \quad 0 \leqslant x \leqslant x_1, \\ U_1(x,y_1) = 0, \quad 0 \leqslant x \leqslant x_1, \\ U_1(0,y) = 0, \quad 0 \leqslant y \leqslant y_1, \\ \left.\dfrac{\partial U_1(x,y)}{\partial x}\right|_{x=x_1} = 0, \quad 0 \leqslant y \leqslant y_1. \end{cases} \tag{5}$$

3. Solution of the problem. The function $U_1(x,y)$ is a solution of the boundary value problem (5) for the Poisson equation and with the variable separation method can be represented as an expansion in terms of eigenfunctions in the variable y with functional coefficients $v_m(x)$ [9–11]:

$$U_1(x,y) = \sum_{m=1}^{\infty} v_m(x)\sin\alpha_m y, \quad \alpha_m = \frac{\pi m}{y_1}. \tag{6}$$

Substituting (6) into the Poisson equation (5), as a result of transformations, it can be obtain that the functions $v_m(x)$ are the solution to the system of second-order ordinary differential equations with constant coefficients:

$$v_m''(x) - (\alpha_m)^2\, v_m(x) = -\frac{2}{\varepsilon_0\, y_1}\int_0^{y_1} \rho(x,y)\sin\alpha_m y\, dy \tag{7}$$

and boundary conditions

$$v_m(0) = 0, \quad \left.\frac{dv_m(x)}{dx}\right|_{x=x_1} = 0.$$

The solution of system (7) makes it possible to find the potential distribution in the entire region of the emission system.

The electrostatic potential distribution $U(x,y)$ has the form:

- $0 \leqslant x \leqslant x_0$

$$U(x,y) = U_0\frac{y}{y_1} + \frac{2}{\pi\,\varepsilon_0}\sum_{i=1}^{N}\tau_i\sum_{m=1}^{\infty}\frac{\sin\left(\alpha_m\, y_{0_i}\right)}{m}\frac{\cosh\left(\alpha_m(x_1-x_0)\right)}{\cosh\left(\alpha_m x_1\right)}\times$$
$$\times\sinh\left(\alpha_m\, x\right)\sin\left(\alpha_m\, y\right); \tag{8}$$

- $x_0 \leqslant x \leqslant x_1$

$$U(x,y) = U_0\frac{y}{y_1} + \frac{2}{\pi\,\varepsilon_0}\sum_{i=1}^{N}\tau_i\sum_{m=1}^{\infty}\frac{\sin\left(\alpha_m\, y_{0_i}\right)}{m}\frac{\sinh\left(\alpha_m x_0\right)}{\cosh\left(\alpha_m x_1\right)}\times$$
$$\times\cosh\left(\alpha_m\left(x_1-x\right)\right)\sin\left(\alpha_m\, y\right); \tag{9}$$

• $x_1 \leqslant x \leqslant x_2 - x_0$

$$U(x,y) = U_0 \frac{y}{y_1} + \frac{2}{\pi\,\varepsilon_0} \sum_{i=1}^{N} \tau_i \sum_{m=1}^{\infty} \frac{\sin(\alpha_m\, y_{0_i})}{m} \frac{\sinh(\alpha_m x_0)}{\cosh(\alpha_m x_1)} \times \\ \times \cosh(\alpha_m (x_1 - x_2 + x)) \sin(\alpha_m\, y)\,; \tag{10}$$

• $x_2 - x_0 \leqslant x \leqslant x_2$

$$U(x,y) = U_0 \frac{y}{y_1} + \frac{2}{\pi\,\varepsilon_0} \sum_{i=1}^{N} \tau_i \sum_{m=1}^{\infty} \frac{\sin(\alpha_m\, y_{0_i})}{m} \frac{\cosh(\alpha_m (x_1 - x_0))}{\cosh(\alpha_m x_1)} \times \\ \times \sinh(\alpha_m (x_2 - x)) \sin(\alpha_m\, y)\,; \tag{11}$$

$$\alpha_m = \frac{\pi m}{y_1}. \tag{12}$$

4. Results of numerical calculations. In accordance with the obtained analytical solution (8)–(12) of the boundary problem (1), the potential distribution was calculated for different values of x_0, which specifies the distance between two emitters.

Figures 2, *a*—*d* show the equipotential distributions throughout the emission diode systems for the following parameter values: $x_1 = 30$, $x_2 = 60$, $y_1 = 10$, $N = 10$, $y_{0_i} = y_1 \dfrac{i}{2N}$, $\tau = -40$, $\tau_i = \tau \dfrac{i}{N}$ $(i = \overline{1, N})$. All geometric parameters and electrode potentials are given in non-dimensional quantities.

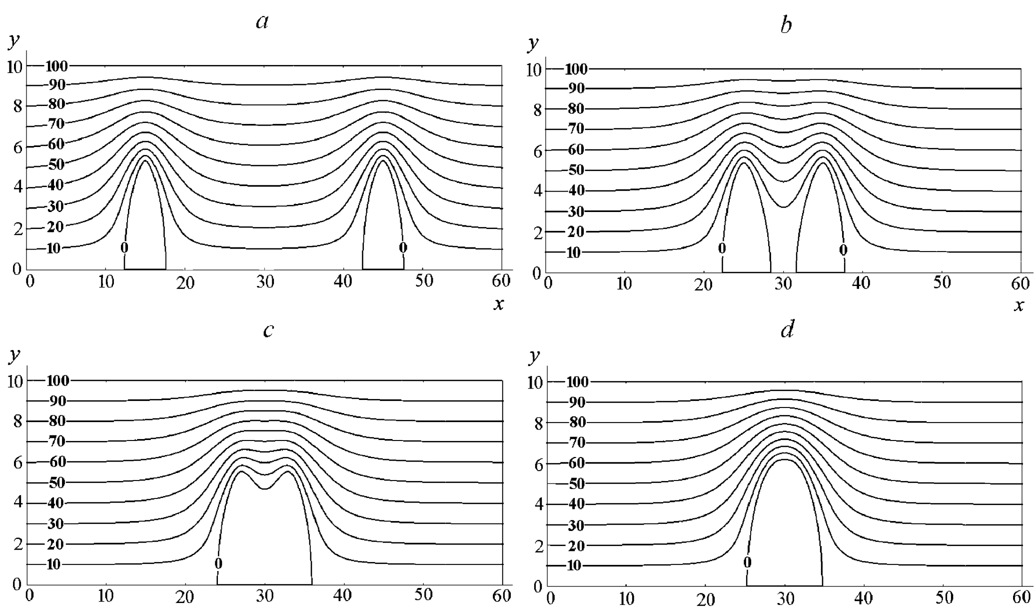

Figure 2. Distribution of equipotentials at $x_0 = 15\,(a)$, $x_0 = 25\,(b)$, $x_0 = 27\,(c)$ and $x_0 = 29\,(d)$

The height of the emitters L can be calculated from condition $U(x_0, L) = 0$.

5. Conclusion. In this article a two-dimensional emission diode system is modeled. The field cathode is a system of two emitters of uniform shape, located on a flat substrate, and with a flat anode parallel to the substrate. To calculate the electrostatic potential distribution, the influence of emitters on the electric field is replaced by the influence of charged filaments system. The variables separation method in Cartesian coordinates is used to solve the boundary problem (1)–(3). The potential distribution is found in the form of Fourier expansions with known coefficients throughout the domain of the emission diode system — formulas (8)–(12). According to formulas obtained in explicit form, the equipotential distributions were presented in graphs, showing the dependence of electrostatic field on the distance between the emitters. All the geometric dimensions of the system are the parameters of the problem.

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Received: September 7, 2022.
Accepted: October 12, 2023.

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Математическое моделирование полевой эмиссионной системы с двумя эмиттерами*

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Для цитирования: *Egorov N. V., Vinogradova E. M., Kurbatova G. I., Klemeshev V. A.* Field emission system with two emitters mathematical modeling // Вестник Санкт-Петербургского университета. Прикладная математика. Информатика. Процессы управления. 2023. Т. 19. Вып. 4. С. 443–448. https://doi.org/10.21638/11701/spbu10.2023.402

Рассмотрена математическая модель двумерной эмиссионной диодной системы с двумя полевыми эмиттерами на плоской подложке, анодом служит плоскость, параллельная плоскости подложки. Согласно предложенной методике, реальный полевой катод совпадает с виртуальным, форма которого определяется нулевой эквипотенциальной поверхностью. Влияние каждого из полевых эмиттеров на распределение электростатического потенциала заменяется влиянием конечного числа заряженных нитей. Решение граничной задачи для уравнения Пуассона найдено в аналитическом виде. Распределение потенциала во всей области исследуемой эмиссионной системы представлено в виде разложений по собственным функциям. Коэффициенты рядов вычислены в явном виде. Приведенные графики демонстрируют зависимость распределения потенциала от расстояния между эмиттерами. Все геометрические размеры системы являются параметрами задачи.

Ключевые слова: микро- и наноэлектроника, полевой эмиттер, математическое моделирование, распределение электростатического потенциала, граничная задача.

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* Исследование выполнено за счет гранта Российского научного фонда № 23-29-00937, https://rscf.ru/project/23-29-00937/