ИНФОРМАТИКА

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Blade-like field cathode with a dielectric coating mathematical modeling

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In this paper the results of a two-dimensional diode emission system based on a blade-like field cathode in a polar coordinate system modeling is presented. The top of the emitter is a circle with a dielectric coating. The anode is a circle coaxial to the top of the emitter. The boundary condition of the first kind is set on the cathode, and the first and second kind on the anode. The problem of calculating the electrostatic potential distribution is reduced to solving a system of linear algebraic equations with constant coefficients. All the geometric dimensions of the system and the values of the potentials on the electrodes are the parameters of the problem.

Keywords: micro- and nanoelectronics, field emitter, mathematical modeling, electrostatic potential distribution, boundary-value problem.

1. Introduction. Over the past few decades, the use and modeling of devices with cathodes based on field emitters has significantly increased in vacuum micro- and nanoelectronics. Such emitters, as a rule, have a small radius of curvature at the apex, which differs by several orders of magnitude in comparison with other geometric parameters of the emission system. Due to such advantages high brightness and low power consumer, the field cathodes are widely used in microscopy, lithography, microwave amplifiers, and X-ray generators [1–3]. To increase the life cycle of field cathodes, the emitter coatings are studied using various materials, including dielectrics [4–9]. To increase the emission current in these systems, so-called blade-like field cathodes are used, which have a much larger emission area compared to single emitters [10].

This work is devoted to the two-dimensional diode emission system with a blade-like field cathode mathematical modeling in a polar coordinate system (r, φ) (Figure). The

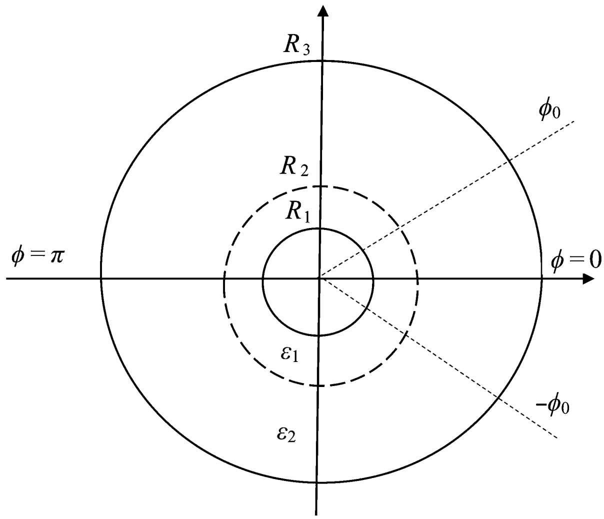

Figure. Schematic representation of the diode field emission system

emitter's top is a circle with a dielectric coating. The anode is a circle coaxial to the emitter top.

2. Mathematical model. The problem parameters: $r = R_1$ ($\varphi \in [-\pi, \pi]$) — emitter's top surface; $r = R_3$ ($\varphi \in [\varphi_0, \pi - \varphi_0]$) — anode surface; $r = R_3$ ($\varphi \in [-\varphi_0, \varphi_0]$) — emitter foundation surface; $r = R_2$ ($\varphi \in [-\pi, \pi]$) — boundary between two dielectrics with the dielectric permittivities ε_1 and ε_2; V_1 — emitter's top boundary condition; $\left.\frac{\partial U(r,\varphi)}{\partial r}\right|_{r=R_3} = Q$ — emitter foundation boundary condition; V_2 — anode boundary condition.

The electrostatic potential distribution $U(r, \varphi)$ is a solution of the Laplace's equation for the boundary-value problem:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2} = 0 \tag{1}$$

and boundary conditions

$$\begin{cases} U(R_1, \varphi) = V_1, & \varphi \in [-\pi, \pi], \\ U(R_3, \varphi) = V_2, & \varphi \in [\varphi_0, \pi - \varphi_0], \\ \left.\dfrac{\partial U(r,\varphi)}{\partial r}\right|_{r=R_3} = Q, & \varphi \in [-\varphi_0, \varphi_0], \\ \varepsilon_1 \left.\dfrac{\partial U(r,\varphi)}{\partial r}\right|_{r=R_2-0} = \varepsilon_2 \left.\dfrac{\partial U(r,\varphi)}{\partial r}\right|_{r=R_2+0}, & \varphi \in [-\pi, \pi]. \end{cases} \tag{2}$$

Due to the fact that the potential distribution function $U(r, \varphi)$ for the entire region ($r \in [R_1, R_3]$, $\varphi \in [-\pi, \pi]$) is symmetric with respect to planes $\varphi = 0$ and $\varphi = \pi$, the problem (1), (2) can be solved for ($r \in [R_1, R_3]$, $\varphi \in [0, \pi]$).

3. Solution of the problem. To solve the boundary-value problem (1), (2) the diode system internal area ($r \in [R_1, R_3]$, $\varphi \in [0, \pi]$) can be divide into following subdomains:

(1) — ($r \in [R_1,\, R_2]\,, \quad \varphi \in [0,\, \varphi_0]$);
(2) — ($r \in [R_2,\, R_3]\,, \quad \varphi \in [0,\, \varphi_0]$);
(3) — ($r \in [R_1,\, R_2]\,, \quad \varphi \in [\varphi_0,\, \pi]$);
(4) — ($r \in [R_2,\, R_3]\,, \quad \varphi \in [\varphi_0,\, \pi]$).

Then

$$U(r,\varphi) = \begin{cases} U_1(r,\varphi), & r \in [R_1,\, R_2]\,, \quad \varphi \in [0,\, \varphi_0]\,, \\ U_2(r,\varphi), & r \in [R_2,\, R_3]\,, \quad \varphi \in [0,\, \varphi_0]\,, \\ U_3(r,\varphi), & r \in [R_1,\, R_2]\,, \quad \varphi \in [\varphi_0,\, \pi]\,, \\ U_4(r,\varphi), & r \in [R_2,\, R_3]\,, \quad \varphi \in [\varphi_0,\, \pi]\,. \end{cases} \tag{3}$$

Let

$$\begin{aligned} V_1(r) &= Q\,R_3\,\frac{\varepsilon_2}{\varepsilon_1}\,\ln(r/R_1) + V_1, \\ V_2(r) &= Q\,R_3\left(\ln(r/R_2) + \frac{\varepsilon_2}{\varepsilon_1}\,\ln(R_2/R_1)\right) + V_1, \\ V_3(r) &= \frac{\varepsilon_2}{\varepsilon_1}\,\frac{(V_2 - V_1)\ln(r/R_1)}{\frac{\varepsilon_2}{\varepsilon_1}\ln(R_2/R_1) + \ln(R_3/R_2)} + V_1, \\ V_4(r) &= \frac{(V_2 - V_1)\ln(r/R_3)}{\frac{\varepsilon_2}{\varepsilon_1}\ln(R_2/R_1) + \ln(R_3/R_2)} + V_2. \end{aligned} \tag{4}$$

The functions $V_1(r)$, $V_2(r)$, $V_3(r)$, $V_4(r)$ (4) obey the Laplace's equation and the boundary conditions (2) for **(i)**-th subdomain accordingly.

The potential distribution (3) $U_i(r,\varphi)$ (i =1, 2, 3, 4) with the variable separation method can be represented for **(i)**-th subdomain in the form [11, 12]:

$$\begin{aligned} U_1(r,\varphi) = V_1(r) &+ \sum_{n=1}^{\infty} a_n e^{-\lambda_n(\varphi_0 - \varphi)}\,\frac{1 + e^{-2\lambda_n\varphi}}{1 + e^{-2\lambda_n\varphi_0}}\,\sin\left(\lambda_n \ln(r/R_1)\right) + \\ &+ \sum_{m=0}^{\infty} c_m\,(r/R_2)^{\mu_m}\,\frac{1 - (r/R_1)^{-2\mu_m}}{1 - (R_1/R_2)^{2\mu_m}}\cos(\mu_m\varphi), \end{aligned} \tag{5}$$

$$\begin{aligned} U_2(r,\varphi) = V_2(r) &+ \sum_{k=0}^{\infty} b_k e^{-\nu_k(\varphi_0 - \varphi)}\,\frac{1 + e^{-2\nu_k\varphi}}{1 + e^{-2\nu_k\varphi_0}}\,\sin\left(\nu_k \ln(r/R_2)\right) + \\ &+ \sum_{m=0}^{\infty} c_m\,(r/R_2)^{-\mu_m}\,\frac{1 + (r/R_3)^{2\mu_m}}{1 + (R_2/R_3)^{2\mu_m}}\cos(\mu_m\varphi), \end{aligned} \tag{6}$$

$$\begin{aligned} U_3(r,\varphi) = V_3(r) &+ \sum_{n=1}^{\infty}\left(a_n + \frac{2(-1)^n}{\lambda_n}\frac{\varepsilon_2}{\varepsilon_1}\left(\frac{T_2 - T_1}{\frac{\varepsilon_2}{\varepsilon_1}\ln(R_2/R_1) + \ln(R_3/R_2)} - Q\,R_3\right)\right) \times \\ &\times\, e^{-\lambda_n(\varphi - \varphi_0)}\,\frac{1 + e^{-2\lambda_n(\pi - \varphi)}}{1 + e^{-2\lambda_n(\pi - \varphi_0)}}\,\sin\left(\lambda_n \ln(r/R_1)\right) + \\ &+ \sum_{p=0}^{\infty} d_p\,(r/R_2)^{\eta_p}\,\frac{1 - (r/R_1)^{-2\eta_p}}{1 - (R_1/R_2)^{2\eta_p}}\cos(\eta_p(\pi - \varphi)), \end{aligned} \tag{7}$$

$$
\begin{aligned}
U_4(r,\varphi) = V_4(r) + \sum_{t=1}^{\infty} g_t\, e^{-\xi_t(\varphi-\varphi_0)}\, \frac{1+e^{-2\xi_t(\pi-\varphi)}}{1+e^{-2\xi_t(\pi-\varphi_0)}} \sin\left(\xi_t \ln\left(r/R_2\right)\right) + \\
+ \sum_{p=0}^{\infty} d_p\, (r/R_2)^{-\eta_p}\, \frac{1-(r/R_3)^{2\eta_p}}{1-(R_2/R_3)^{2\eta_p}} \cos(\eta_p(\pi-\varphi)),
\end{aligned} \tag{8}
$$

where

$$
\begin{gathered}
\lambda_n = \frac{\pi\, n}{\ln\left(R_2/R_1\right)}, \qquad \mu_m = \frac{(2m+1)\,\pi}{2\varphi_0}, \qquad \nu_k = \frac{(2k+1)\,\pi}{2\ln\left(R_3/R_2\right)}, \\
\eta_p = \frac{(2p+1)\,\pi}{2(\pi-\varphi_0)}, \qquad \xi_t = \frac{\pi\, t}{\ln\left(R_3/R_2\right)}.
\end{gathered} \tag{9}
$$

The functions (5)–(9) $U_i(r,\varphi)$ $(i = 1,\, 2,\, 3,\, 4)$ as the potential distribution (3) $U(r,\varphi)$, written in the form of series in eigenfunctions, satisfies boundary conditions (2) on the cathode and anode surfaces. These formulas provide the next potential distribution continuity conditions:

$$
\begin{aligned}
&U_1(R_2,\varphi) = U_2(R_2,\varphi), && \varphi \in [0,\, \varphi_0]\,; \\
&U_1(r,\varphi_0) = U_3(r,\varphi_0), && r \in [R_1,\, R_2]\,; \\
&U_3(R_2,\varphi) = U_4(R_2,\varphi), && \varphi \in [\varphi_0,\, \pi]\,; \\
&U_2(r,\varphi_0) = U_4(r,\varphi_0), && r \in [R_2,\, R_3]\,.
\end{aligned}
$$

To calculate the unknown coefficients a_n, b_k, c_m, d_p, g_t the additional potential distribution continuity conditions can be used:

$$
\begin{aligned}
&U_2(r,\varphi_0) = U_4(r,\varphi_0), && r \in [R_2,\, R_3]\,, \\
&\varepsilon_1 \left.\frac{\partial U_1(r,\varphi)}{\partial r}\right|_{r=R_2} = \varepsilon_2 \left.\frac{\partial U_2(r,\varphi)}{\partial r}\right|_{r=R_2}, && \varphi \in [0,\, \varphi_0]\,, \\
&\left.\frac{\partial U_1(r,\varphi)}{\partial \varphi}\right|_{\varphi=\varphi_0} = \left.\frac{\partial U_3(r,\varphi)}{\partial \varphi}\right|_{\varphi=\varphi_0}, && r \in [R_1,\, R_2]\,, \\
&\varepsilon_1 \left.\frac{\partial U_3(r,\varphi)}{\partial r}\right|_{r=R_2} = \varepsilon_2 \left.\frac{\partial U_4(r,\varphi)}{\partial r}\right|_{r=R_2}, && \varphi \in [\varphi_0,\, \pi]\,, \\
&\left.\frac{\partial U_2(r,\varphi)}{\partial \varphi}\right|_{\varphi=\varphi_0} = \left.\frac{\partial U_4(r,\varphi)}{\partial r}\right|_{\varphi=\varphi_0}, && r \in [R_2,\, R_3]\,.
\end{aligned} \tag{10}
$$

The continuity conditions (10) and the eigenfunctions systems orthogonality lead to the linear algebraic equations system with respect to unknowns coefficients a_n, b_k, c_m, d_p, g_t:

$$b_k \frac{1}{2} \ln (R_3/R_2) - (-1)^k \sum_{t=1}^{\infty} g_t \frac{(-1)^t \xi_t}{\nu_k^2 - \xi_t^2} = \\ = \frac{1}{\nu_k} \left(\frac{\varepsilon_2}{\varepsilon_1} \ln (R_2/R_1) + \frac{1}{\nu_k} (-1)^k \right) \left(\frac{V_2 - V_1}{\frac{\varepsilon_2}{\varepsilon_1} \ln (R_2/R_1) + \ln (R_3/R_2)} - Q R_3 \right), \tag{11}$$

$$(-1)^m \sum_{n=1}^{\infty} a_n \frac{(-1)^n \lambda_n}{\lambda_n^2 + \mu_m^2} - (-1)^m \frac{\varepsilon_2}{\varepsilon_1} \sum_{k=0}^{\infty} b_k \frac{\nu_k}{\nu_k^2 + \mu_m^2} + \\ + c_m \frac{\varphi_0}{2} \left(\coth (\mu_m (R_2/R_1)) + \frac{\varepsilon_2}{\varepsilon_1} \coth (\mu_m (R_3/R_2)) \right) = 0, \tag{12}$$

$$a_n \frac{1}{2} \ln (R_2/R_1) (\text{ th } (\lambda_n \varphi_0) + \text{ th } (\lambda_n (\pi - \varphi_0))) + (-1)^n \sum_{m=0}^{\infty} c_m \frac{(-1)^m \mu_m}{\lambda_n^2 + \mu_m^2} + \\ + (-1)^n \sum_{p=0}^{\infty} d_p \frac{(-1)^p \eta_p}{\lambda_n^2 + \eta_p^2} = - \text{ th } (\lambda_n (\pi - \varphi_0)) \times \\ \times \frac{2(-1)^n}{\lambda_n} \frac{\varepsilon_2}{\varepsilon_1} \left(\frac{T_2 - T_1}{\frac{\varepsilon_2}{\varepsilon_1} \ln (R_2/R_1) + \ln (R_3/R_2)} - Q R_3 \right), \tag{13}$$

$$(-1)^p \sum_{n=1}^{\infty} a_n \frac{(-1)^n \lambda_n}{\lambda_n^2 + \eta_p^2} + d_p \frac{\pi - \varphi_0}{2} \left(\coth (\eta_p (R_2/R_1)) + \frac{\varepsilon_2}{\varepsilon_1} \coth (\eta_p (R_3/R_2)) \right) - \\ - (-1)^p \frac{\varepsilon_2}{\varepsilon_1} \sum_{t=1}^{\infty} g_t \frac{\xi_t}{\xi_t^2 + \eta_p^2} = -2 (-1)^p \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2 + \eta_p^2} \times \\ \times \frac{\varepsilon_2}{\varepsilon_1} \left(\frac{T_2 - T_1}{\frac{\varepsilon_2}{\varepsilon_1} \ln (R_2/R_1) + \ln (R_3/R_2)} - Q R_3 \right), \tag{14}$$

$$- \sum_{m=0}^{\infty} c_m \frac{\mu_m}{\mu_m^2 + \nu_k^2} + b_k \frac{1}{2} \ln (R_3/R_2) \text{ th } (\nu_k \varphi_0) + \\ + \sum_{p=0}^{\infty} d_p \frac{(-1)^p \eta_p}{\nu_k} \left(\frac{\eta_p}{\eta_p^2 + \nu_k^2} \frac{(-1)^k}{\text{sh } (\eta_p (\ln (R_3/R_2)))} - \frac{\nu_k}{\eta_p^2 + \nu_k^2} \right) + \\ + (-1)^k \sum_{t=1}^{\infty} g_t \frac{\xi_t}{\nu_k} \frac{(-1)^t \xi_t}{(\nu_k^2 - \xi_t^2)} \text{ th } (\xi_t (\pi - \varphi_0)) = 0. \tag{15}$$

4. Conclusion. This article presents the results of mathematical modeling the two-dimensional diode emission system with a blade-like field cathode system. The emitter top has a dielectric coating. To solve the boundary problem (1), (2) the variable separation method in polar coordinates was used. The electrostatic potential distribution is presented in terms of eigenfunctions (3)–(9), and the definition of unknown coefficients in potential expansions is reduced to solving a linear algebraic equations system (11)–(15). All the geometric dimensions of the system and the values of the potentials on the electrodes are the parameters of the problem.

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Математическое моделирование полевого катода лезвийной формы с диэлектрическим покрытием

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В работе представлены результаты моделирования двумерной диодной эмиссионной системы на основе полевого катода лезвийной формы в полярной системе координат. Вершина эмиттера представляет собой окружность, на которую нанесено диэлектрическое покрытие. Анод — окружность, коаксиальная вершине эмиттера. На катоде задано граничное условие первого рода, на аноде — первого и второго рода. Задача вычисления — распределение электростатического потенциала — сведена к решению системы линейных алгебраических уравнений с постоянными коэффициентами. Все геометрические размеры системы и потенциалы на электродах являются параметрами задачи.

Ключевые слова: микро- и наноэлектроника, полевой эмиттер, математическое моделирование, распределение электростатического потенциала, граничная задача.

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